Period 4, May 20, 2025

32. $|\tan\theta| \leq 1$

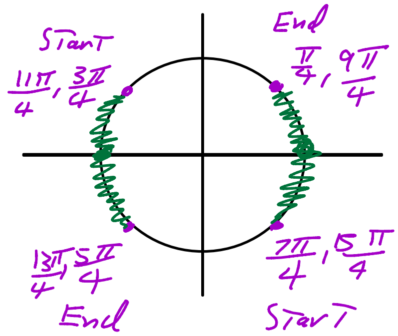
$-1 \leq \tan\theta \leq 1$

$\tan\frac{\pi}{4} = 1$ $\quad -1 \leq \phi \leq 1$ False

$\tan\frac{3\pi}{4} = -1$ $\quad -1 \leq 0 \leq 1$ True

$\tan\frac{5\pi}{4} = 1$

$\tan\frac{7\pi}{4} = -1$



Test

$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

$\tan\frac{\pi}{2} = \frac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}} = \frac{1}{0} = \phi$

$\tan\pi = 0$

$\tan\frac{3\pi}{2} = \phi$

Start $\leq \theta \leq$ End

$\underbrace{\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}}_{\frac{4\pi}{4}}, \frac{15\pi}{4}$ $\qquad$ $\cancel{\frac{\pi}{4}}, \frac{5\pi}{4}, \underbrace{\frac{9\pi}{4}, \frac{13\pi}{4}}_{\frac{4\pi}{4}}$

Can't End before you Start

$\theta = \frac{3\pi}{4} + \frac{4\pi}{4}(n-1)$ $\qquad$ $\theta = \frac{5\pi}{4} + \frac{4\pi}{4}(n-1)$

$\frac{3\pi}{4} + \frac{4\pi n}{4} - \frac{4\pi}{4}$ $\qquad$ $\frac{5\pi}{4} + \frac{4\pi n}{4} - \frac{4\pi}{4}$

$\frac{(4k+3)\pi}{4} \leq \theta \leq \frac{(4k+5)\pi}{4}$

$k=0$

$\frac{3\pi}{4} \leq \theta \leq \frac{5\pi}{4}$

$k=1$

$\frac{7\pi}{4} \leq \theta \leq \frac{9\pi}{4}$

$-\frac{\pi}{4} + \frac{4\pi n}{4} \leq \theta \leq \frac{\pi}{4} + \frac{4\pi n}{4}$

$n=1$

$\frac{3\pi}{4} \leq \theta \leq \frac{5\pi}{4}$

$n=2$

$\frac{7\pi}{4} \leq \theta \leq \frac{9\pi}{4}$

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23. $1 - \sin^2\theta = \cos\theta$

$\cos^2\theta = \cos\theta$

$-\cos\theta \qquad -\cos\theta$

$\cos^2\theta - \cos\theta = 0$

$\cos\theta(\cos\theta - 1) = 0$

$\cos\theta = 0$ or $\cos\theta - 1 = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $\cos\theta = 1$

$\theta = 0, 2\pi$

Group

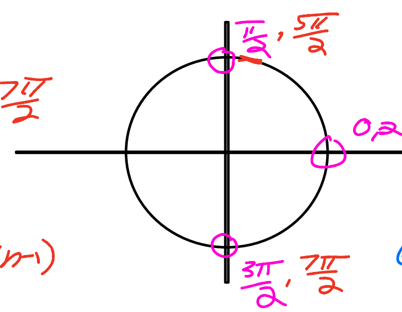
$\underbrace{\frac{\pi}{2}, \frac{3\pi}{2}}_{\frac{2\pi}{2}}, \underbrace{\frac{5\pi}{2}}_{\frac{2\pi}{2}}, \underbrace{\frac{7\pi}{2}}_{\frac{2\pi}{2}}$

$\frac{\pi}{2} + \frac{2\pi}{2}(n-1)$

$\frac{\pi}{2} + \frac{2\pi n}{2} - \frac{2\pi}{2}$

$\frac{2\pi n - \pi}{2}$

$\theta = \frac{\pi}{2}(2n-1)$



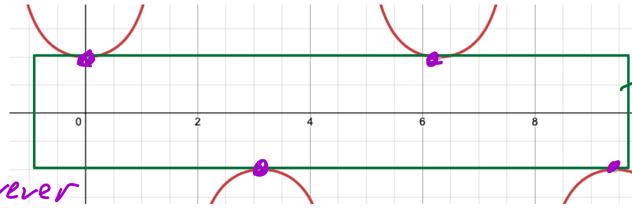
$\theta = \underbrace{0, 2\pi}_{2\pi}, \underbrace{4\pi}_{2\pi}$

$0 + 2\pi(n-1)$

$\theta = 2\pi n - 2\pi$

$\theta = 2\pi k$

35 $|\sec\theta| \le 1$



no Real

$-1 \le \sec\theta \le 1$

$-1 < \sec\theta < 1$ never happens

but

$\sec 0 = 1$ $\quad\theta = 0, \pi, 2\pi, 3\pi$

$\sec\pi = -1$ $\quad\theta = \pi n$

$\sec 2\pi = 1$

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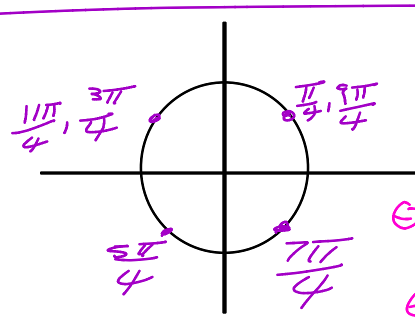
31. $\tan^2\theta = 1$

$\tan\theta = \pm 1$

$\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\tan\theta = -1 \Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$



$a_n = a_1 + d(n-1)$

$\frac{\pi}{4} + \frac{2\pi}{4}(n-1)$

$\theta = \frac{\pi}{4} + \frac{2\pi n}{4} - \frac{2\pi}{4}$

$\theta = -\frac{\pi}{4} + \frac{2\pi n}{4}$

$\theta = \underbrace{\frac{\pi}{4}, \frac{3\pi}{4}}_{2\pi/4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ $\quad 2\pi/4$

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28. $2\cos^2\theta - \sin\theta - 1 = 0$

$2(1 - \sin^2\theta) - \sin\theta - 1 = 0$

$2 - 2\sin^2\theta - \sin\theta - 1 = 0$

$-2\sin^2\theta - \sin\theta + 1 = 0$ $\quad$ or $\quad 2\sin^2\theta + \sin\theta - 1 = 0$

$-2 \cdot 1 = -2$ $\qquad 2 \cdot -1 = -2$

$-2 + 1 = -1$ $\qquad 2 - 1 = 1$

$2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$

$2\sin\theta(\sin\theta + 1) - 1(\sin\theta + 1) = 0$

$(\sin\theta + 1)(2\sin\theta - 1) = 0$

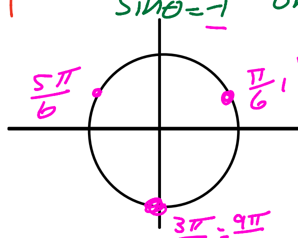
$\sin\theta = -1$ or $\sin\theta = \frac{1}{2}$

$-2\sin^2\theta - 2\sin\theta + 1\sin\theta + 1 = 0$

$-2\sin\theta(\sin\theta + 1) + 1(\sin\theta + 1) = 0$

$(\sin\theta + 1)(-2\sin\theta + 1) = 0$

$\sin\theta = -1$ or $\sin\theta = \frac{1}{2}$



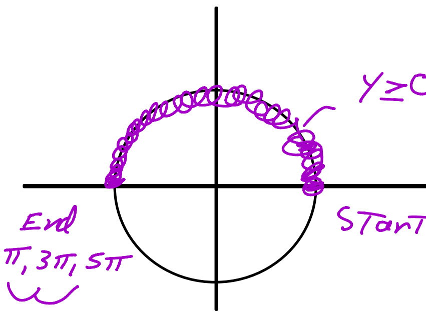
$\theta = \frac{3\pi}{2}$ and $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{13\pi}{6}$

$\frac{4\pi}{6} \quad \frac{4\pi}{6} \quad \frac{4\pi}{6} = d$

34. $\sin\theta \geq 0$

$y \geq 0$



$y \geq 0 \Rightarrow \sin\theta \geq 0$

End

$\pi, 3\pi, 5\pi$

$\underbrace{\quad}_{2\pi \quad 2\pi}$

$\pi(2n+1)$

Start $0, 2\pi, 4\pi$

$\underbrace{\quad}_{2\pi \quad 2\pi}$

$2\pi n$

$$2\pi n \leq \theta \leq \pi(2n+1)$$

Express the repeating decimal as a fraction in lowest terms.

$$0.\overline{46} = \underbrace{\frac{46}{100} + \frac{46}{10{,}000}}_{\frac{1}{100}} + \frac{46}{1{,}000{,}000} + \cdots$$

$\frac{1}{100} = r$

$$\frac{a_1}{1-r} = \frac{\frac{46}{100}}{1-\frac{1}{100}} = \frac{\frac{46}{100}}{\frac{100}{100}-\frac{1}{100}} = \frac{\frac{46}{100}}{\frac{99}{100}} = \frac{46 \cdot \cancel{100}}{\cancel{100} \cdot 99} = \frac{46}{99}$$

The sequence given is defined using a recursion formula. Write the first four terms of the sequence.

$a_1 = 4$ and $a_n = 4a_{n-1}$ for $n \geq 2$

$a_2 = 4a_{2-1} = 4 \cdot a_1 = 4 \cdot 4 = 16$

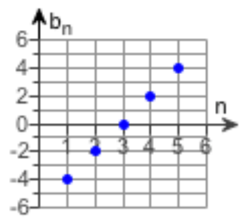
$a_3 = 4a_{3-1} = 4 \cdot a_2 = 4 \cdot 16 = 64$

$a_4 = 4a_{4-1} = 4 \cdot a_3 = 4 \cdot 64 = 256$

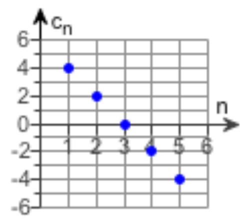
Use the graphs of $\{b_n\}$ and $\{c_n\}$ to find the indicated sum.

$$\sum_{i=1}^{5} (2b_i + c_i)$$

The graph of $\{b_n\}$



The graph of $\{c_n\}$



$(2b_1+c_1)+(2b_2+c_2)+(2b_3+c_3)+$
$(2b_4+c_4)+(2b_5+c_5)$

$(2(-4)+4)+(2\cdot -2+2)+(2\cdot 0+0)$
$+(2\cdot 2+-2)+(2(4)+-4)=$

$-4+-2+0+2+4=0$

$b_1=-4$
$b_2=-2$
$b_3=0$
$b_4=2$
$b_5=4$

$c_1=4$
$c_2=2$
$c_3=0$
$c_4=-2$
$c_5=-4$

Express the sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

$n=1 \quad 2 \quad 3 \quad n$

$$\frac{1}{4}+\frac{2}{5}+\frac{3}{6}+\cdots+\frac{13}{13+3}$$

$1+3 \quad 2+3 \quad 3+3 \qquad n=13$

$$\frac{1}{4}+\frac{2}{5}+\frac{3}{6}+\cdots+\frac{13}{13+3}=\sum_{\boxed{1}}^{\boxed{13}} \boxed{\frac{n}{n+3}}$$

You are considering two job offers. Company A will start you at $30,000 a year and guarantee a raise of $3000 per year. Company B will start you at a higher salary, $40,000 a year, but will only guarantee a raise of $700 per year. Find the total salary that each company will pay over a ten-year period. Which company pays the greater total amount?

Company A $= \underbrace{30{,}000+33{,}000}_{+3000}+\underbrace{36{,}000}_{+3000}+\cdots$ Year 10 (57,000)

$a_n = 30{,}000+3000(n-1) = 27{,}000+3000n \qquad a_{10}=57{,}000$

$\text{Sum}=\dfrac{n(a_1+a_n)}{2}=\dfrac{10(87{,}000)}{2}$

$= 435{,}000$

Company B $= \underbrace{40{,}000+40{,}700}_{+700}+\underbrace{41{,}400}_{+700=d}+\ldots$ Year 10

$a_n = 40{,}000+700(n-1) \Rightarrow a_n = 39{,}300+700n \qquad a_{10}=39{,}300+7000=46{,}300$

$\text{Sum}=\dfrac{10(40{,}000+46{,}300)}{2}=\dfrac{10(86{,}300)}{2}=431{,}500$

Company A will pay $ 435000 over a ten-year period.

Company B will pay $ 431500 over a ten-year period.

Company A pays the greater total amount.

Use the formula for the general term (the nth term) of a geometric sequence to find the indicated term of the following sequence with the given first term, a_1, and common ratio, r.

Find a_{10} when $a_1 = 3$ and $r = 2$.

$a_n = a_1 \cdot r^{n-1}$

$a_n = 3 \cdot (2)^{n-1}$

$a_{10} = 3 \cdot (2)^{10-1} = 3 \cdot 2^9 = 3 \cdot 512 = 1536$

Suppose you save $1 the first day of a month, $4 the second day, $16 the third day, and so on. That is, each day you save four times as much as you did the day before. What will you put aside for savings on the eighth day of the month?

1, 2, 3, 4, 5 ...

$\underbrace{1, 4, 16, 64}_{\cdot 4 \ \cdot 4 \ \cdot 4} \ldots$ $r = 4$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 1 \cdot 4^{n-1}$

$a_8 = 1 \cdot 4^{8-1} = 4^7$

$a_8 = 16,384$

Find the sum of the first 11 terms of the geometric sequence. Use the formula for the sum of the first n terms of a geometric sequence.

5, −20, 80, −320, ... $n = 11$

$\cdot -4 \ \cdot -4 \ \cdot -4 = r$

$Sum = \frac{a_1(1-r^n)}{1-r}$

$a_1 = 5$

$Sum = \frac{5(1-(-4)^n)}{1-(-4)} = \frac{\cancel{5}(1-(-4)^n)}{\cancel{5}} = 1-(-4)^n = 1-(-4)^{11}$

$1-(-4194304)$

$1+4194304$

4194305

Write a formula for the general term of the geometric sequence. Then use the formula for a_n to find a_7, the seventh term of the sequence.

$a_1 = 18$

$\underbrace{18, 6,}_{\div 3} \underbrace{\; 2,}_{\div 3} \frac{2}{3}, \ldots$ $\quad r = \frac{1}{3}$

$a_n = a_1 (r)^{n-1}$

$a_n = 18\left(\frac{1}{3}\right)^{n-1}$

$a_7 = 18\left(\frac{1}{3}\right)^{7-1} = 18 \cdot \frac{1^6}{3^6} = \frac{18}{729} = \frac{9 \cdot 2}{9 \cdot 81} = \frac{2}{81}$

If $\{a_n\}$ and $\{b_n\}$ equal the following, find $a_9 + b_9$.

$\{a_n\} = -6, 12, -24, 48, \ldots$ $\quad -2 \quad -2 \quad r = -2$ $\quad a_n = -6(-2)^{n-1}$

$\{b_n\} = 5, -2, -9, -16, \ldots$ $\quad a_n = a_1 + d(n-1)$

$-7 \; -7 \; -7 = d$ $\quad a_n = 5 - 7(n-1)$

$a_9 = -6(-2)^{9-1} = -6(-2)^8 = -6(256) = -1536$

$b_9 = 5 - 7(9-1) = 5 - 7(8) = 5 - 56 = -51$

$a_9 + b_9$

$-1536 + -51 = -1587$

As n increases, the terms of the sequence $a_n = \left(1 + \frac{1}{n}\right)^n$ get closer and closer to the number e (where $e \approx 2.7183$). Use a calculator to find a_{10}, a_{100}, a_{1000}, $a_{10,000}$, and $a_{100,000}$, comparing these terms to your calculator's decimal approximation for e.

$a_{10} \approx$ 2.5937

$a_{100} \approx$ 2.7048

$a_{1000} \approx$ 2.7169

$a_{10,000} \approx$ 2.7181

$a_{100,000} \approx$ 2.7183

(Round to four decimal places as needed.)

$a_{100} = \left(1 + \frac{1}{100}\right)^{100} = (1 + 0.01)^{100} = (1.01)^{100}$

$a_{100000} = \left(1 + \frac{1}{100{,}000}\right)^{100{,}000} = (1 + 0.00001)^{100{,}000} = (1.00001)^{100{,}000}$

Find the indicated sum.

$$\sum_{i=1}^{5} \frac{(-1)^{i+1}}{(i+1)!} = \frac{(-1)^{1+1}}{(1+1)!} + \frac{(-1)^{2+1}}{(2+1)!} + \frac{(-1)^{3+1}}{(3+1)!} + \frac{(-1)^{4+1}}{(4+1)!} + \frac{(-1)^{5+1}}{(5+1)!} = \frac{1}{2} + \frac{-1}{6} + \frac{1}{24} + \frac{-1}{120} + \frac{1}{720}$$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

Write a formula for the general term (the nth term) of the arithmetic sequence $a_1, a_2, a_3, a_4, \ldots$. Then use the formula for a_n to find a_{20}, the 20th term of the sequence.

$4, -1, -6, -11, \ldots$

$-5\ -5\ -5 = d$

$a_n = a_1 + d(n-1)$

$a_n = 4 + -5(n-1)$

$a_n = 4 - 5n + 5$

$a_n = 9 - 5n$

$a_{20} = 9 - 5(20) = 9 - 100 = -91$